18.704, FINAL PAPER INFORMATION

Writing math papers is more like writing literature than you may think! It is not just about writing down the nitty gritty proofs just to show you can. You want your peers to be able to read your paper, enjoy reading it, and learn something from it. To that end, you want to craft a narrative and tell a cohesive story that is clear, compelling, and emphasizes the key features of the mathematics.

It is important to start early so you can properly digest and give your paper time to mature! Your paper will be written in pairs. It should be around 15 pages, typed in LaTeX. Key deadlines are listed below, but I highly encourage staying ahead of the schedule.

- **Friday, March 12**: Submit paper topic preferences – rank your top 3 paper topic choices. If two of you agree beforehand to work together, both of you should email me confirming that you would like to work with the other person and specify your paper topic preferences.
- **Friday, March 26**: Submit typed proposal/description/abstract of your paper with main literature sources identified.
- **Friday, April 9**: Submit at least 5 typed pages of your paper and an outline of the entire paper.
- **Friday, April 30**: Submit complete draft of your paper.
- **Friday, May 20**: Final paper due.

Abstract. Include an abstract at the beginning of your paper. If you are using the amsart document class to write your paper, this can be done by using the abstract environment.

Definitions. Define "everything" that the paper uses, meaning if any terms or concepts could be unclear to you or your classmates, you should define them.

(a) If the paper/reference you are using is less than 15 pages in length, expanding on definitions and certain topics addressed by the paper you are using as a guide will help you on producing a 15 page long final paper.
(b) If the paper/reference you are using as a guide is longer than 15 pages then you should still make sure that in your final paper definitions are given for the convenience of the audience. In this case though you will have to decide on which of the topics addressed in the paper you are using as a guide you want to concentrate.

Introduction. Your paper should begin with a clear introduction.

(a) The introduction is where you begin building the story arc, so you need to set the scene by providing motivation for the problem you will explain, previous work on the subject, why it is interesting.
(b) You should introduce the main ideas, definitions, and concepts.
(c) You should clearly define the purpose(s) of the paper.
(d) The introduction is often the *only* section of the paper a casual reader will actually read, so you need to provide an "executive summary" that clearly explains what the paper is about and how it is relevant.
(e) Unlike in a story, you *should* say the punchline in the introduction and not save it for the end.

Focus. If the paper you are considering is very long, you should only focus on a subset (perhaps one or two) of the topics considered in the paper so that the paper you write is an appropriate length (15 pages).

Audience. You should be reasonable with how deep you delve into specific topics.

(a) Remember that your target audience is your classmates.
(b) You should not go so deeply into a specific topic so that when you give your talk nobody else understands what you are doing.

Plagiarism. Loosely, plagiarism is writing someone else's work off as your own; you should avoid doing this. Below are some ways to avoid plagiarism, but consult the handout on plagiarism for more detailed information.

(a) Citing classical results such as the Fundamental Theorem of Algebra by name (and not referring back to the original work) is okay.
(b) Less well-known results should be cited.
(c) For talks, set up the questions in a historical context and refer to the works you used to solve the problem; if you do not do this it makes it seem like everything you showed was your original work.
(d) Cite all sources used to prepare the paper in a References section.
(e) Explain the main references in the introduction. For example you could write: Section 2 of this paper is based on [ref]. If later on in the paper you need to recall a specific theorem in that reference [ref], please indicate the number of the theorem in [ref]. In this way people reading your paper don't have to go through the whole reference.
(f) Do not put verbatim theorems, definitions, and other technical statements in quotation marks.
(g) Verbatim non-technical statements should be in quotes.
(h) You could cite a theorem by writing: Theorems 2 and 3 are due to XYZ of [ref].

Suggested topics. Please Google or Wikipedia these topics to get a sense of what they are about! Or ask someone who knows something about the topic! Avoid using Wikipedia as a primary source, but it is a useful learning tool and helpful as a starting point and pointer to other references.

I have included possible sources for where to get the material, but **the listed sources are not required or necessary and may not be sufficient**. The idea is that at least it's a place to start, and you can supplement with other stuff you find on the Internet.

(A) Topic of your choosing. If you have another topic in mind for your final paper, you must:
  - Find another student who's excited to learn and write on the subject with you.
  - Pitch the topic to me (including possible references)
(B) Poincaré-Birkhoff-Witt (PBW) Theorem
  - Kirillov, Theorem 5.11 and the references therein
(C) The Lie algebra $\mathfrak{g}_2$ and its representations
  - Fulton & Harris, Sections 22.1–22.3
(D) Gelfand–Tsetlin basis for $\mathfrak{gl}_n$
  - `https://arxiv.org/pdf/math/0211289.pdf`
(E) Classification of semisimple Lie algebras by Dynkin diagrams
  - Fulton & Harris, Section 21

(F) Relation between representation theory and particle physics: spherical Laplace operator and the hydrogen atom
- Kirillov, Introduction and Section 4.9

(G) Relation between representation theory and particle physics: Clebsch–Gordan coefficients
- `https://en.wikipedia.org/wiki/Clebsch%E2%80%93Gordan_coefficients`

(H) Representations of $\mathfrak{sl}_2(\mathbb{F})$ where $\mathbb{F}$ is algebraically closed of characteristic $p > 2$.
- `https://gauss.math.yale.edu/~il282/RT/RT4.pdf`

(I) Schur–Weyl duality
- Etingof, Section 5.18
- `https://en.wikipedia.org/wiki/SchurWeyl_duality`

(J) Modular forms as representations of $\mathfrak{gl}_2(\mathbb{C})$.
- `https://www.math.uchicago.edu/~emerton/gl2.pdf`
- `https://en.wikipedia.org/wiki/Representation_theory_of_SL2(R)`

(K) Beilinson–Bernstein localization: relation of representations of $\mathfrak{sl}_2(\mathbb{C})$ to $\mathcal{D}$-modules (differential operators on $\mathbb{P}^1$).
- requires some algebraic geometry (or commutative algebra)