18.704: SEMINAR IN ALGEBRA, SPRING 2021 Representation theory of Lie algebras via examples

Instructor: Jonathan Wang, jpwng@mit.edu The best way to contact me is by email (I will try to respond within 24 hours).

 $Office\ Hours:$ Monday 6-7 pm Eastern, Wednesday 6-7 pm Eastern, or by appointment. Join via Canvas \rightarrow Zoom.

Mathematical communications specialist: Susan Ruff, ruff@mit.edu

Office Hours: Highly recommended to schedule a one-on-one appointment here. There are also drop-in office hours on Fridays 1-2 pm Eastern time via this Zoom link

Class Meetings: Our official class time is Tuesday/Thursday 1:00p - 2:30p via Zoom.

- All Zoom meetings during official class times will be automatically recorded.
- Please turn on your video during each class!
- Students will work in pairs to prepare and deliver presentations. Most class meetings will consist of one or two student presentations followed by either discussions or a recap by me.

For the presenters:

- You can present by handwriting live using something like Notability, OneNote, etc. or by using slides (made with Beamer, Google Slides, Powerpoint, etc.) or a combination of both. If you are handwriting live, it is still recommended to "pre-write" an outline of what you'll say and then fill in the blanks, for the purposes of neatness and time management.
- For the final project presentation you are required to make slides (using Beamer, Google Slides, Powerpoint, or something else).
- At the start of your presentation, please share your outline/slides to Zoom chat so everyone else can have a local copy to scroll through. After class, I will upload these slides to Canvas for future reference.
- The presenters will write part of the problem set for the week: before your presentation, send me a .tex and .pdf file with around 4-6 problems (2-3 problems if 25 minute presentation) that will go on the next problem set. These can be technical details or proofs that you didn't want to work out during your presentation (but you will also have to submit this pset!).

For each presentation, the following things will work in tandem:

- Two (2) students will present their talk.
- Each non-presenting student (14) will handwrite comments on the presentation. After class, each student will fill out a presentation critique online (via Google Forms—these will be sent out later). These comments are to help the presenter improve, so be thoughtful, kind, and constructive in your comments! Click here for a sample critique.
- Some advice on presenting can be found here.
- Before each meeting, **all** students should have read the sections of the textbook to be presented.
- For the first round of presentations, each pair **must** schedule a practice presentation with me and Susan beforehand. For later presentations, you are also encouraged to schedule practice presentations with me or Susan.

Textbooks: We will be applying a much more general theory of Lie algebra representations to particular examples, so we will "pick and choose" material from the following textbooks:

- Representation Theory: A First Course (Fulton and Harris)
- An Introduction to Lie Groups and Lie Algebras (Kirillov, Jr.)
- Introduction to Representation Theory (Etingof et al.)
- Representations of Semisimple Lie Algebras in the BGG Category O (Humphreys) We will only use this book as a last resort, and only in particular examples, so do not be put off by how intimidating it looks!

PDFs of all of the textbooks are available on Canvas.

Pre-requisites: 18.701 or (18.06 and 18.703) or (18.700 and 18.703). Students should already be comfortable with writing proofs and comfortable with linear algebra in the sense of abstract vector spaces. Some familiarity with rings and modules will be helpful, but will also be reviewed as we progress. If any of these topics are mysterious to you, learn them before the semester starts and get help in office hours as needed. Also, problem sets and the term paper must be prepared using $IAT_{E}X$; for help with this, see *More math into LaTeX* (Grätzer) or this online resource. Also, this can be useful if you're looking for a particular symbol.

Website: Canvas

Discord. For informal discussions, there is a Discord page (link on Canvas). This is mainly for discussions among yourselves -I will sometimes check in to answer questions if you @mention me, but email is still the best way to reach me.

Problem sets: Weekly, due Fridays by 11:59pm EST/EDT, via upload to Gradescope. Late submissions will not be accepted.

- Solutions should be written in IAT_EX .
- You should not expect to be able to solve every single problem on your own; instead you are encouraged to discuss questions with each other or to come to office hours, so that when you submit an assignment you are pretty sure that it is complete and correct. If you meet with a study group, you will learn best if you do as many problems as you can on your own before meeting. Write-ups must be done independently. (In practice, this means that it is OK for other people to explain their solutions to you, but you must not be looking at other people's solutions as you write your own.)
- *Highly recommended*: Use https://psetpartners.mit.edu/ to form groups to work on problem sets with!
- There will be a Zoom "PSet Room" that you can join at any time to work on problem sets with other people.
- At the top of each problem set you submit, write either the text "Sources consulted: none" or a list of all sources consulted other than the listed textbooks (also include the listed textbooks if you are using material not covered). This is required. (Examples of things that should be listed if consulted: a classmate, a tutor, a friend, a website, a textbook, solutions from a previous semester, etc.)

Final paper. You will write one expository paper on a topic of your choice. Suggested topics will be distributed soon after the start of the semester. Papers are expected to be around 15 pages in length, to be written in pairs.

Grading: Attendance is mandatory. Any absence needs to be pre-approved by S3 or the instructor. Absences without explanation will affect the final grade. Here is the breakdown:

- Problem sets (30%)
- Written paper (30%)
- Presentations (40%) = presentations on textbook material (20%) + your final presentation on your written paper (15%) + written critiques of peer presentations (5%).

There will be no exams.

Important Dates: Key dates (subject to change) for your final paper are:

- Friday, March 12: Submit paper topic preferences.
- Friday, March 26: Submit typed proposal/description/abstract of your paper with main literature sources identified.
- Friday, April 9: Submit at least 5 typed pages of your paper.
- Friday, April 30: Submit complete draft of your paper.
- Tuesday, May 11: Peer feedback session on paper drafts during class time.
- Thursday, May 20, 10 PM EDT: Final paper due.

General important dates:

- Friday, March 19: Add Deadline
- Thursday, April 29: Drop Deadline

If you need disability accommodations: Please speak with Kathleen Monagle, Associate Dean in Student Disability Services (SDS) via email or call 617-253-1674, ideally before the semester begins. If you have a disability but do not plan to use accommodations, it is still recommended that you meet with SDS staff to familiarize yourself with the services and resources of the office. If you have already been approved for accommodations, please send the accommodation letter to Theresa Cummings (617-253-4977) early in the semester.